Degrees of freedom

The number of degrees of freedom of a dynamical system is defined as the total number of co-ordinates or independent ~~variables~~ quantities required to describe completely the position and configuration of the system.

Configuration means the arrangement of the constituent elements of the system in space.

Regarding the position of the system, let a particle moves along a straight line i.e along x-axis, then its position can be specified by its displacement along the x-axis.

Therefore such a particle has one translational degree of freedom. If the particle moves along a surface or a plane, it has two degrees of freedom because two co-ordinates are needed to specify its position. Therefore, it has two degrees of freedom.

If the particle moves in space, its position at any instant can be determined by three co-ordinates X, Y & Z. Therefore, such a particle has three translational degrees of freedom. In case of rigid body there will be six degrees of freedom in space three due to translatory motion and there will be three due to rotatory motion. If the system is made up of two particles then each particle has three degrees of freedom. If the distance between the particles remain constant then there is one definite relation between them and hence the no. of degrees of freedom will be $6 - 1 = 5$.

It means that

No. of degrees of freedom of a system is described

by the relation —

$$N = 3A - R$$

Where N = No. of degrees of freedom of a system

A = No. of particles in the system.

R = number of independent relations among the particles.

(a) Monoatomic gas

Monoatomic gas molecule (like neon, argon, helium) has only ~~only~~ one atom. It has translatory motion only and hence it has three degrees of freedom.

Also, putting, $A = 1$, $R = 0$,

then $N = 3 \times 1 - 0 = 3$.

(b) Di-atomic gas

The molecule of a diatomic gas like hydrogen, Oxygen, nitrogen etc has two atoms.

Here, $A = 2$, $R = 1$

$N = 3 \times 2 - 1$

$= 5$ degrees of freedom

Triatomic gases: It has three atoms, for ex. SO_2, H_2S etc.

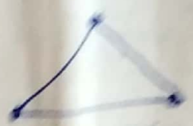
In a linear molecule, two atoms lie on either side of the central atom.

In this case, $A = 3$, $R = 2$

 (a)

then $N = 3 \times 3 - 2 = 7$ degrees of freedom.

In a non-linear molecule,

 (b)

there are three independent relations among the three atoms.

$A = 3$, $R = 3$

$N = 3 \times 3 - 3 = 6$ degrees of freedom.

Law of equipartition of energy

The average kinetic energy associated with each degree of freedom $= \frac{1}{2} KT$.

Determination of γ from the degrees of freedom

Let a polyatomic gas molecule has n-degrees of freedom.

Total energy associated with a gram-

... of the gas ...

$E = n \times \frac{1}{2} kT \times N$

$= \frac{n}{2} RT$

$C_V = \frac{dE}{dT}$

$\therefore C_V = \frac{d}{dT}(E) = \frac{d}{dT}\left(\frac{n}{2}RT\right) = \frac{n}{2}R$

As, $C_p = C_V + R$

$C_p = \frac{n}{2}R + R = \left(\frac{n}{2}+1\right)R$

As, $\gamma = \frac{C_p}{C_V}$

$\therefore \gamma = \frac{\left(\frac{n}{2}+1\right)\cancel{R}}{\frac{n}{2}\cancel{R}} = \frac{2}{n}\left(\frac{n}{2}+1\right)$

$\boxed{\gamma = 1 + \frac{2}{n}}$

$\frac{1}{2}mc^2 = \frac{3}{2}kT$

$c^2 = u^2 + v^2 + w^2$

As x, y and z are all equivalent, mean of square velocities along the three axes are equal.

$\frac{1}{2}mc^2 = 3\left[\frac{1}{2}mu^2\right] = \frac{3}{2}kT$

$\frac{1}{2}mu^2 = \frac{1}{2}kT$

haphazard — mere chance (आकस्मिक)

Rigorous — clear visible

Average kinetic energy associated with each degree of freedom is $\frac{1}{2}kT$.